جامعة بنها

كلية الحاسبات والذكاء الإصطناعي

Discrete Mathematics

Lecture 03

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Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

Negating Quantified Expressions (1/4)

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$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

"Every student in your class has taken a course in calculus"

Negating Quantified Expressions (2/4)

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"There is at least one student in your class who has not taken a course in calculus"

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$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

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$\exists x \boldsymbol{P}(\boldsymbol{x})$ **:**

"At least one student in your class has taken a course in calculus"

The negation of this statement is

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$$\neg\exists \boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}) \equiv \forall \boldsymbol{x}\neg\boldsymbol{P}(\boldsymbol{x})$$

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

"If you have a current password, then you can log onto the network."

"You have a current password."

Therefore,

"You can log onto the network."

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

"If you have a current password, then you can log onto the network."

"You have a current password."

Premises

Therefore,

"You can log onto the network."

Conclusion

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

$p \rightarrow q$

p **Premises**

$\therefore q$ **Conclusion**

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$p \rightarrow q$

p **Premises**

―――――――

$\therefore q$ **Conclusion**

This argument is valid if $\big((p \rightarrow q) \land p\big) \rightarrow q$ is a tautology.

Valid Arguments in Propositional Logic

An **argument** in propositional logic is a sequence of propositions. All the proposition in the argument are called **premises** and the final proposition is called the **conclusion**.

$p \rightarrow q$

p **Premises**

$\therefore q$ **Conclusion**

This argument is valid if $((p \rightarrow q) \land p) \rightarrow q$ is a tautology.

Rules of Inference (2/9)

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

$p \rightarrow q$

p

$\therefore q$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1

p	q	$p \rightarrow q$				
T	T	T				
T	F	F				
F	T	T				
F	F	T				

$p \rightarrow q$

p

$\therefore q$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2

p	q	$p \to q$	p			
T	T	T	T			
T	F	F	T			
F	T	T	F			
F	F	T	F			

$$p \to q$$
$$p$$
$$\overline{\qquad\qquad}$$
$$\therefore q$$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2

p	q	$p \to q$	p	$(p \to q) \wedge p$		
T	T	T	T	T		
T	F	F	T	F		
F	T	T	F	F		
F	F	T	F	F		

$p \to q$

p

———————

$\therefore q$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2 Conclusion

p	q	$p \to q$	p	$(p \to q) \land p$	q	
T	T	T	T	T	T	
T	F	F	T	F	F	
F	T	T	F	F	T	
F	F	T	F	F	F	

$$p \to q$$
$$p$$
$$\overline{}$$
$$\therefore q$$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2 Conclusion

p	q	$p \rightarrow q$	p	$(p \rightarrow q) \wedge p$	q	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	F	F	F	T

$p \rightarrow q$

p

$\therefore q$

$((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology

Rules of Inference (3/9)

TABLE 1 Rules of Inference. Part 1

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Rules of Inference (3/9)

TABLE 1 Rules of Inference. Part 2

Rule of Inference	*Tautology*	*Name*
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Rules of Inference (4/9)

Example1

Using the truth table to show that the hypotheses

$p \vee q$

$\neg p \vee r$

lead to the conclusion

$q \vee r$

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (4/9)

Example1

Using the truth table to show that the hypotheses

$p \vee q$

$\neg p \vee r$

$q \vee r$

Premise 1 ($p \vee q$), Premise 2 ($\neg p \vee r$), Conclusion ($q \vee r$)

p	q	r	$p \vee q$	$\neg p$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
T	T	T	T	F	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	F

Rules of Inference (5/9)

Example2

Using the rules of inference to show that the hypotheses

$\neg p \wedge q$

$r \rightarrow p$

$\neg r \rightarrow s$

$s \rightarrow t$

lead to the conclusion

t

Rules of Inference (5/9)

Example2

$\neg p \land q$

$\therefore \neg p$

$p \land q$ $\therefore p$	$(p \land q) \rightarrow p$	Simplification

~~$\neg p \land q$~~

$r \rightarrow p$

……………………………………

$\neg r \rightarrow s$

$s \rightarrow t$

Rules of Inference (5/9)

Example2

$$\neg p \land q$$
$$\therefore \neg p$$

$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification

$$\neg p$$
$$r \to p$$
$$\therefore \neg r$$

$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens

~~$\neg p \land q$~~

~~$r \to p$~~

$\neg r \to s$

$s \to t$

Example2

$\neg r$

$\neg r \rightarrow s$

$\therefore s$

p $p \rightarrow q$ $\therefore q$	$(p \land (p \rightarrow q)) \rightarrow q$	Modus ponens

~~$\neg p \land q$~~

~~$r \rightarrow p$~~

~~$\neg r \rightarrow s$~~

$s \rightarrow t$

Rules of Inference (5/9)

Example2

$\neg r$

$\neg r \rightarrow s$

$\therefore s$

p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens

s

$s \rightarrow t$

$\therefore t$

conclusion

~~$\neg p \wedge q$~~

~~$r \rightarrow p$~~

~~$\neg r \rightarrow s$~~

~~$s \rightarrow t$~~

Chapter 2: Basic Structures

- Sets.
- Functions.
- Sequences, and Summations.
- Matrices.

Sets (1/24)

A **set** is an unordered collection of objects.

The objects in a set are called the *elements*, or *members*, of the set. A set is said to contain its elements.

Sets (2/24)

$S = \{a, b, c, d\}$

We write $a \in S$ to denote that a is an element of the set S. The notation $e \notin S$ denotes that e is not an element of the set S.

Sets (3/24)

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots\ , 99\}$.

ellipses (…)

Sets (4/24)

Another way to describe a set is to use **set builder** notation.

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\},$$

$$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}.$$

Sets (5/24)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of all **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of all **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$,
the set of all **rational numbers**

$\mathbf{R}$, the set of all **real numbers**

$\mathbf{R}^+$, the set of all **positive real numbers**

$\mathbf{C}$, the set of all **complex numbers**.

Interval Notation

Closed interval $[a, b]$
Open interval (a, b)

$$[a, b] = \{x \mid a \leq x \leq b\}$$

$$[a, b) = \{x \mid a \leq x < b\}$$

$$(a, b] = \{x \mid a < x \leq b\}$$

$$(a, b) = \{x \mid a < x < b\}$$

Sets (7/24)

If A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$, if A and B are equal sets.

- The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal, because they have the same elements.

- $\{1, 3, 3, 5, 5, 5\}$ is the same as the set $\{1, 3, 5\}$ because they have the same elements.

Empty Set

There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by $\emptyset$.

The empty set can also be denoted by { }

Cardinality

The cardinality is the number of distinct elements in S.
The cardinality of S is denoted by $|S|$.

Example1

$S = \{a, b, c, d\}$
$|S| = 4$

$A = \{1, 2, 3, 7, 9\}$

$\emptyset = \{\ \}$

Example1

$$S = \{a, b, c, d\}$$
$$|S| = 4$$

$$A = \{1, 2, 3, 7, 9\}$$
$$|A| = 5$$

$$\emptyset = \{\ \}$$
$$|\emptyset| = 0$$

Sets (11/24)

Example2

$S = \{a, b, c, d, \{2\}\}$
$|S| =$

$A = \{1, 2, 3, \{2,3\}, 9\}$
$|A| =$

$\{\emptyset\} = \{\{\ \}\}$
$|\{\emptyset\}| =$

Example2

$$S = \{a, b, c, d, \{2\}\}$$
$$|S| = 5$$

$$A = \{1, 2, 3, \{2,3\}, 9\}$$
$$|A| = 5$$

$$\{\emptyset\} = \{\{\ \}\}$$
$$|\{\emptyset\}| = 1$$

Infinite

A set is said to be **infinite** if it is not finite.
The set of positive integers is infinite.

$$Z^+ = \{1,2,3, \dots\}$$

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

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$$(\boldsymbol{A} \subseteq \boldsymbol{B}) \equiv (\boldsymbol{B} \supseteq \boldsymbol{A})$$

$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

Subset

For every set S,

$$(i)\ \emptyset \subseteq S \quad \text{and} \quad (ii)\ S \subseteq S.$$

To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

Proper Subset

The set A is a subset of the set B but that $A \neq B$,
we write $A \subset B$
and say that A is a **proper subset** of B.

$$A \subset B \leftrightarrow \big(\forall x(x \in A \rightarrow x \in B) \land \exists x(x \in B \land x \notin A)\big)$$

Sets (15/24)

Example

For each of the following sets,
determine whether 3 is an element of that set.

$\{1,2,3,4\}$

$\{\{1\},\{2\},\{3\},\{4\}\}$

$\{1,2,\{1,3\}\}$

Example

For each of the following sets,
determine whether 3 is an element of that set.

$3 \in \{1,2,3,4\}$

$3 \notin \{\{1\},\{2\},\{3\},\{4\}\}$

$3 \notin \{1,2,\{1,3\}\}$

Venn Diagram

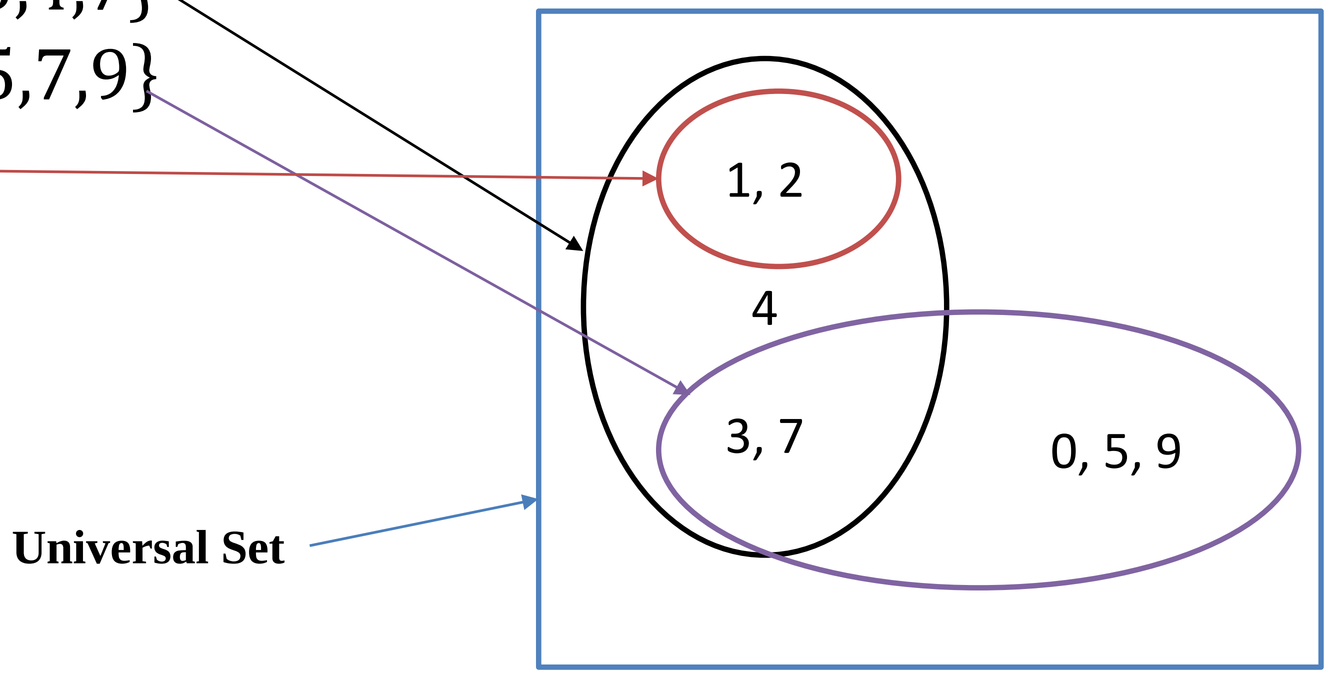
$A = \{1,2,3,4,7\}$
$B = \{0,3,5,7,9\}$
$C = \{1,2\}$

Venn Diagram

$A = \{1,2,3,4,7\}$

$B = \{0,3,5,7,9\}$

$C = \{1,2\}$



Power Set

The set of all subsets.

If the set is S. The power set of S is denoted by $P(S)$.

The number of elements in the power set is $2^{|S|}$

Power Set

The set of all subsets.

If the set is $S.$ The power set of S is denoted by $P(S)$.

The number of elements in the power set is $2^{|S|}$

$S = \{1,2,3\}$

$\boldsymbol{|P(S)| = 2^3 = 8}$ **elements**

$P(S) = 2^S$

$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Example1

What is the power set of the empty set?

Example1

What is the power set of the empty set?

$$\mathcal{P}(\emptyset) = \{\emptyset\}.$$

Example2

What is the power set of the set $\{\emptyset\}$?

Sets (20/24)

Example2

What is the power set of the set $\{\emptyset\}$?

$$\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}.$$

The ordered n-tuple

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, … , and a_n as its nth element.

In particular, ordered 2-tuples are called ordered pairs (e.g., the ordered pairs (a, b))

Cartesian Products

Let A and B be sets.

The **Cartesian product** of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Cartesian Products - Example

Let $A = \{1,2\}$, and $B = \{a, b, c\}$

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

$|A \times B| = |A| * |B| = 2 * 3 = 6$

Cartesian Products - Example

Let $A = \{1,2\}$, and $B = \{a, b, c\}$

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

$|A \times B| = |A| * |B| = 2 * 3 = 6$

Find $B \times A$?

The Cartesian product of more than two sets.

The Cartesian product of the sets $A_1, A_2, \dots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \dots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \dots, n$. In other words,

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \dots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \dots, n\}.$$

Sets (24/24)

Example:

$A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$

$$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}.$$

Set Operations (1/7)

Union

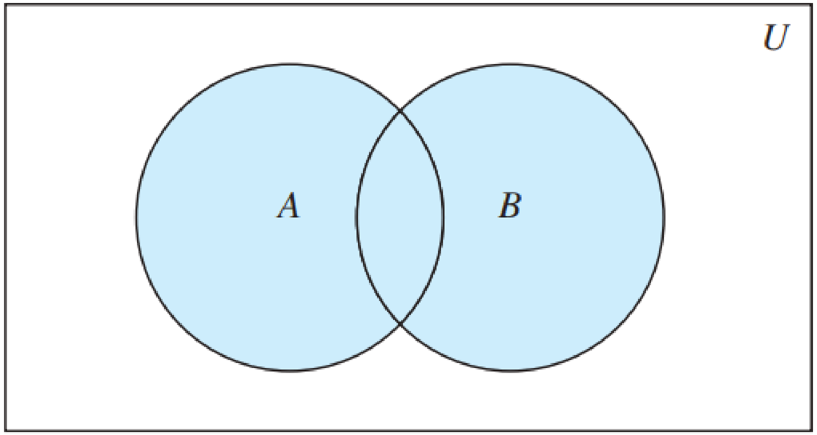
Let A and B be sets. The **union** of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Set Operations (1/7)

Union

Let A and B be sets. The **union** of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.



$A \cup B$ is shaded.

Set Operations (1/7)

Union

Let A and B be sets. The **union** of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.

The union of the sets $\{1,3,5\}$ and $\{1,2,3\}$ is the set $\{1,2,3,5\}$

Set Operations (2/7)

Intersection

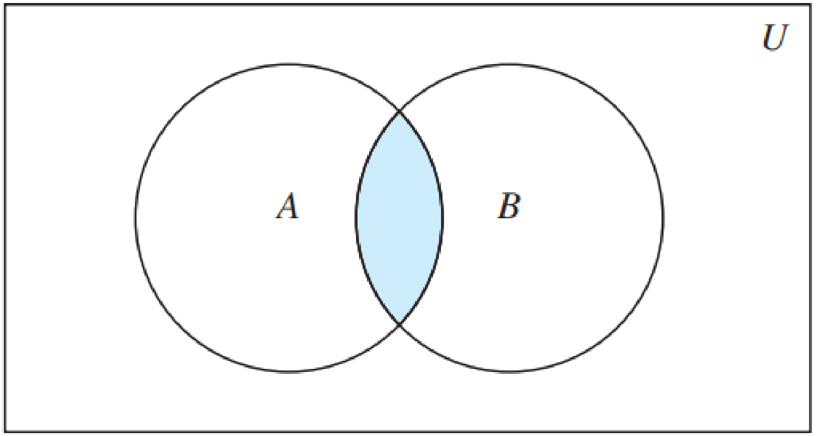
Let A and B be sets. The **intersection** of the sets A and B , denoted by $A \cap B$, is the set that contains those elements that are in both A and B.

$$A \cap B = \{x \mid x \in A \land x \in B\}$$

Set Operations (2/7)

Intersection

Let A and B be sets. The **intersection** of the sets A and B , denoted by $A \cap B$, is the set that contains those elements that are in both A and B.



$A \cap B$ is shaded.

Set Operations (2/7)

Intersection

Let A and B be sets. The **intersection** of the sets A and B , denoted by $A \cap B$, is the set that contains those elements that are in both A and B.

The intersection of the sets $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is the set $\{1, 3\}$

Set Operations (3/7)

Disjoint

Two sets are called **disjoint** if their intersection is the empty set.

$$A \cap B = \emptyset$$

Set Operations (4/7)

Difference

Let A and B be sets. The difference of A and B , denoted by $A - B$, is the set containing those elements that are in A but not in B.

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

Difference

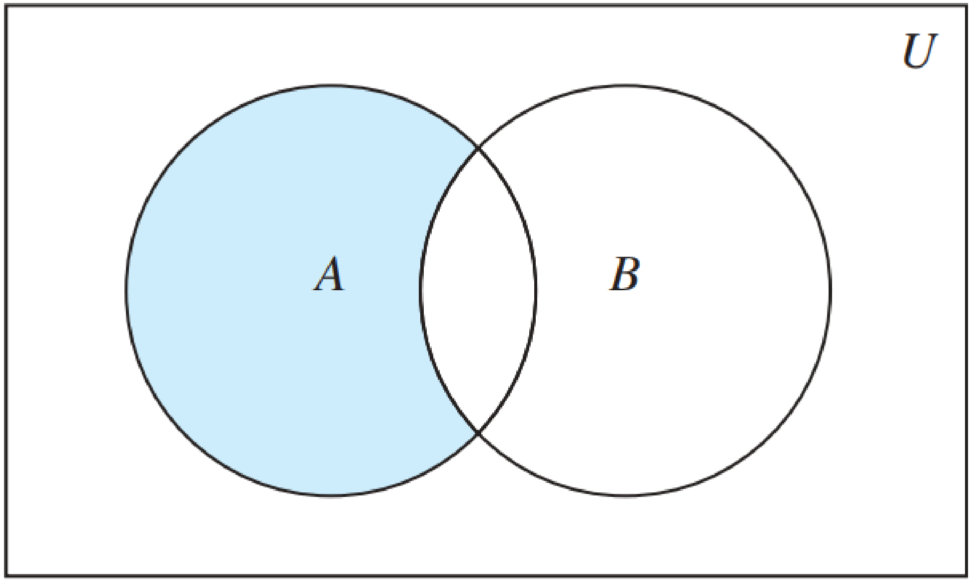
Let A and B be sets. The difference of A and B , denoted by $A - B$, is the set containing those elements that are in A but not in B.

$$A = \{1,3,5\}, \qquad B = \{1,2,3\}$$

$$A - B = \{5\}$$

Set Operations (4/7)

Difference



$A - B$ is shaded.

Set Operations (5/7)

Complement

Let U be the universal set.

The complement of the set A , denoted by $\bar{A}$

An element x belongs to U if and only if $x \notin A$.

$$\bar{A} = \{x \in U \mid x \notin A\}$$

Set Operations (5/7)

Complement

Let U be the universal set.

The complement of the set A , denoted by $\bar{A}$

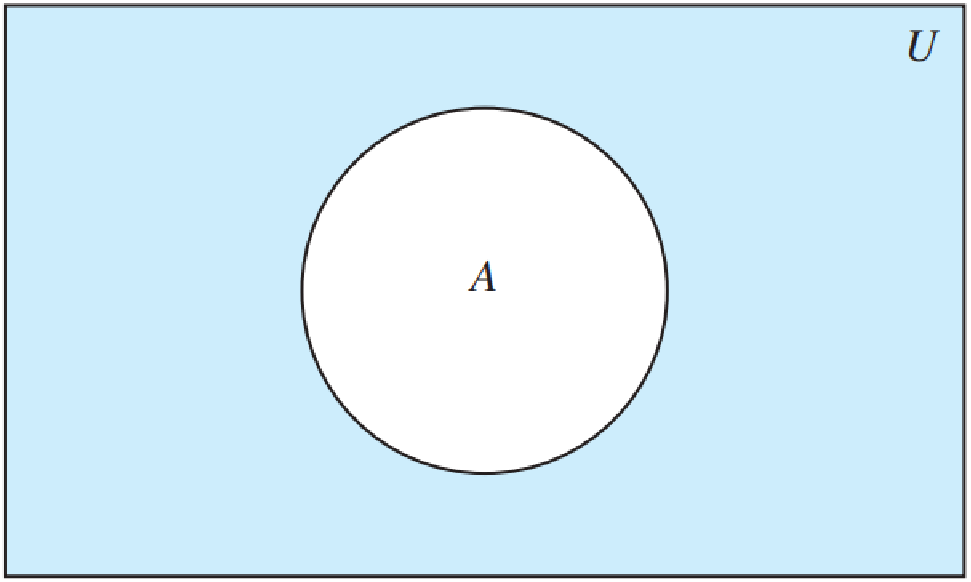
An element x belongs to U if and only if $x \notin A$.

$$U = \{1,2,3,4,5\}, \qquad A = \{1,3\}$$

$$\bar{A} = \{2,4,5\}$$

Set Operations (5/7)

Complement



$\overline{A}$ is shaded.

Set Operations (6/7)

Generalized Unions

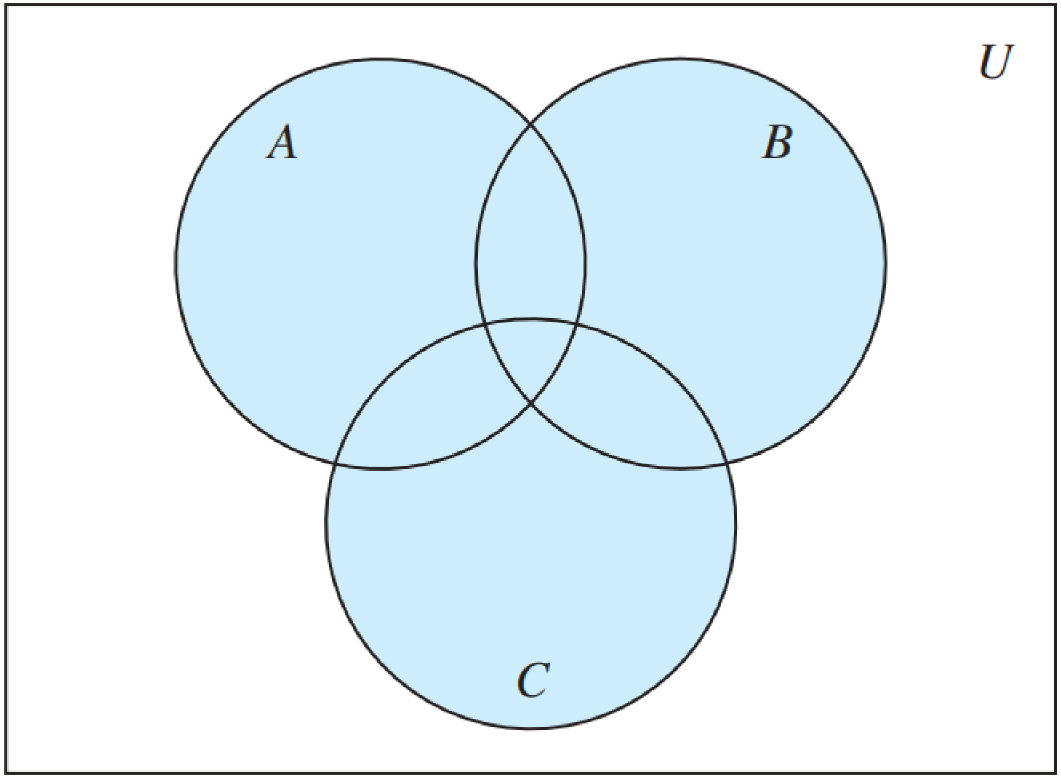
We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Set Operations (6/7)

Generalized Unions



$A \cup B \cup C$ is shaded.

Set Operations (7/7)

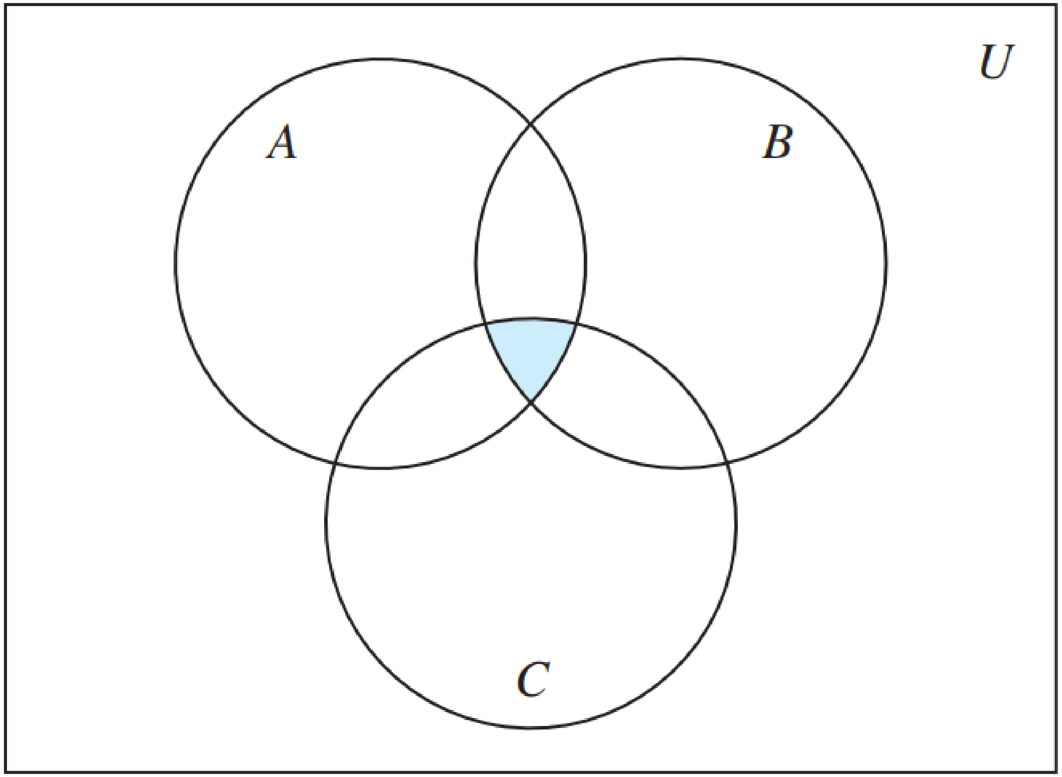
Generalized Intersections

We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Generalized Intersections



$A \cap B \cap C$ is shaded.

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz

Lecture #3: https://www.youtube.com/watch?v=bNNpZa3fwqQ&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=8 Up to time 00:31:18

https://www.youtube.com/watch?v=1FEEjRCWo6E&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=10

https://www.youtube.com/watch?v=RdbOHQddn3Y&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=11

https://www.youtube.com/watch?v=iSuD96uQ2zU&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=12 Up to time 00:12:46

Thank You

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